

# Integrating Certified Lengths to Strengthen Metrology Network Uncertainty

Authors:

Joseph Calkins, PhD  
New River Kinematics  
[joe@kinematics.com](mailto:joe@kinematics.com)

Scott Sandwith  
New River Kinematics  
[scott@kinematics.com](mailto:scott@kinematics.com)

## Abstract

Calibrated and traceable scale lengths add significant value to 3D metrology networks. The primary benefit is to provide an observed physical standard with a documented uncertainty within the measurement process. This leads to an assumption that the object measurements are generally not more precise than the net difference between the calibrated and observed scale bar lengths. A question often raised is: while the observations of the calibrated scale bar are traceable how do they relate to object measurements?

A method for uncertainty analysis of 3D metrology networks with traceable scale standards is presented in this paper. The effect of using the observations on standards as constraints vs. just reporting their differences are studied with an expanded version of the Unified Spatial Metrology Network (USMN) measurement uncertainty analysis tool. A stronger network solution results when a length standard is applied in opportunistic configurations. A three step process is outlined which produces propagated uncertainty estimates, 3D graphical representation, and hardcopy reports of the results.

This study shows the accuracy of calibrated length measurements in a survey do not always relate to general object measurements. Having a method to characterize the measurement process is critical to understanding process variation and overall measurement quality. The geometric dependences between instruments, standards, and the point network dominate measurement uncertainty. Understanding the dependencies and then using them to your advantage is key to successfully producing quality measurement results with reliable measurement uncertainties.

## Introduction

3D metrology system users work to reliably scale their measurements of objects and maintain a traceable link between their results and dimensional standards. One aspect of the scaling process compensates for the effects of the objects thermal expansion or contraction. It is required when objects are measured at temperatures other than the reference temperature. Accurate traceable thermal scaling has traditionally been dependent on measuring the object's actual temperature and using a good estimate for the object's Coefficient of Thermal Expansion (CTE). While this is commonly accepted practice, it introduces considerable uncertainty into measurement results. Integrating calibrated length standards of like material type can improve measurement results and reduce 3D metrology measurement uncertainty.

## Thermal Compensation Scaling for 3D Metrology

When building or verifying objects a reference temperature for the project is defined. It is typically 20° C (68° F.) However objects are seldom measured in the shop at the reference temperature. A thermal compensation scale factor is applied to the measurements to compensate the actual measurements to represent object dimensions when it is at the reference temperature. With measurement results at the reference temperature they are compared against nominal configurations or other measurements of mating assembly components. The process goal enables analysis of assembly components (e.g., fuselages and hull sections) measured at different temperatures to be compared.

The “accuracy” or more appropriately “uncertainty” of the thermal scaling process and its effect on the measurement result can consume significant portions of part tolerances. An error in the scale factor is considered a systematic error and therefore can be cumulative. It is directly proportional to object size so a small error in the thermal compensation scale is magnified when applied to a large part. Confidently and accurately compensating for thermal change is an important element of sound metrology practice.

## Thermal Length Compensation

The dimension change induced by temperature difference is significant. A function to compute the expected dimensional change uses nominal object length, the material’s CTE, and temperature delta from the reference.

The length of the object ( $L_i$ ) at temperatures other than the reference is modeled with the function below.

$$L_i = L_0(1 - \alpha\Delta T)$$

where :

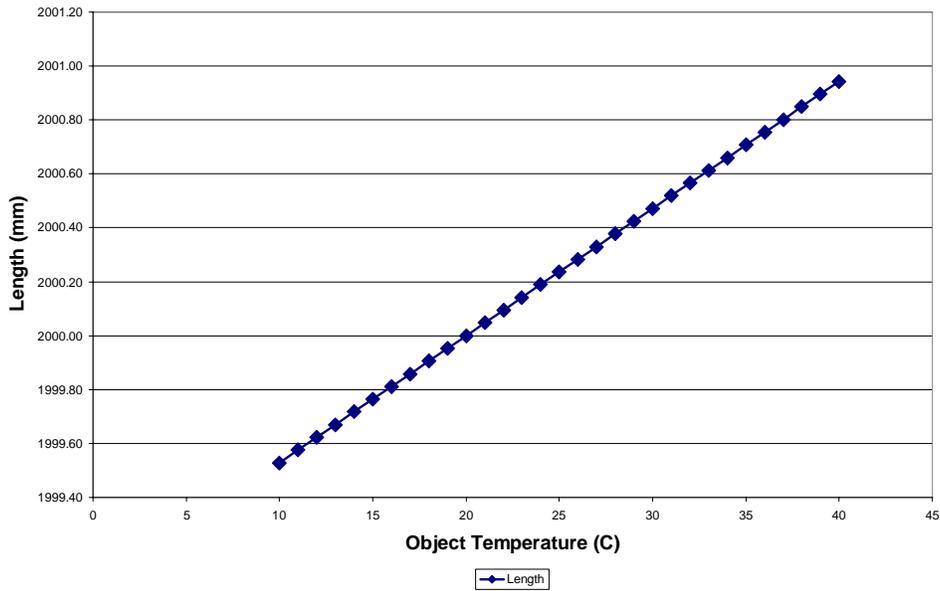
$L_i$  = actual length at temperature

$L_0$  = calibrated length at reference  
temperature

$\alpha$  = CTE for scale bar material (ppm/°C)

$\Delta T$  = temperature delta between reference and actual  
and temperature (°C)

The length of a 2-meter long aluminum object at different temperatures is shown in the graph below.

**Thermal Length Compensation (2 meter Aluminum Scale Bar)**

At the reference temperature the bar is 2-meters long. At 30° C (86° F) the bar length is expected to be 2000.47 mm (78.7587 inches) based on the Thermal Length Compensation function. Changing the bar's temperature by 10° C (18° F) the bar length changes 0.47 mm (0.0186").

## Thermal Compensation Errors

The process and results using the thermal compensation function are straight-forward to implement in 3D metrology software. Understanding the sources of potential error in this process and their effects is significant. There are three components and potential sources of error in the thermal compensation function. The sources of error include object temperature delta from the reference, a material CTE and when using a calibrated length standard for comparison, it's published length uncertainty.

Object temperature is typically measured in one or possibly two locations on the object with a thermocouple device. The unit may or may not be certified depending on measurement process controls. The measurements are generally done on the surface of the object. Good practice suggests using a certified thermocouple or infra-red temperature monitor to make the object temperature measurements.

There are several sources of temperature measurement error in computing the expected length change with thermal compensation function. For the delta temperature input in the function the core/average temperature of the object is needed to get accurate results. An object's surface temperature is generally not the same as its core or average temperature in shop environments. Surface temperatures will typically change faster than core temperature.

Shop environments generally have vertical temperature gradients. Meaning the temperature at the floor level is not the same at the top of the part or tool. The

atmospheric temperature may vary by 5° C within 4 or 5 meters. It makes selecting the optimal location to measure an object's average temperature difficult to determine. Additional complications are apparent because the temperature gradient tends to induce an expansion gradient in the object being measured. Compensating for a thermal expansion gradient across an object is not typical in 3D metrology software. Typical 3D metrology processes ignore potential thermal expansion gradients. The common practice is to make a single temperature measurement at a convenient location on the part/tool surface and assume that it is representative for the entire object.

Certification certificates for typical shop object temperature measurement equipment generally state  $\pm 0.5$  ° C ( $\pm 1$  ° F) uncertainties. The variability between temperature measurement units is significant given the amount of dimensional uncertainty that results when the object temperature vary by this amount. Newer temperature sensors are available with certifications in the range of  $\pm 0.04$  ° C (reference: ScAlert specification by 4G Metrology).

Getting an accurate estimate for an object's material CTE property is important when using the thermal compensation function to adjust the actual measurements to the reference temperature. The published values for particular material vary by alloy and condition. A report by NIST indicates the CTE material property can vary significantly from the published value. <http://emtoolbox.nist.gov/Temperature/Slide14.asp#Slide14>

Material properties references publish average CTE values over temperature ranges. The range of interest for 3D metrology application is about the typical reference temperature of 20°C. However the value at 20 °C is seldom available. The average values are in general, averages over the range 0-100 °C, and the uncertainty of the published values is estimated to be 3-5 % ( $k = 2$ ) over this temperature range.

Large parts and tools are generally not made from a single material type. Different materials each with different CTE's are commonly measured metrology applications. Estimating a CTE for a structure made with different material types can be a significant challenge. When large structures are mounted or secured in the concrete floor the combination becomes more complex when using the thermal compensation function.

## **Certified Scale Length Uncertainty**

When metrology labs certify scale bars a point to point distance is provided. Along with the distance an uncertainty statement for the point to point distance is provided. The uncertainty is based on the calibration process in the laboratory. The measurement method and precision of the instruments used to measure the distance between the points are used to quantify the certification uncertainty.

There are several characteristics about certified scale bars that are of interest. Specifically the distance between the points is now a known quantity at the reference temperature. The scale bar material type is also known. Typically scale bars are produced with the same material as the object being measured in the shop/factory. The uncertainty for the distance between these points is generally small when compared to the part tolerances.

## Checking/Confirming Thermal Compensation

With measurements of object temperature and the material's CTE property, metrology system users scale their measurements of objects at shop temperatures to match the reference temperature. To check or confirm that the scaling process was accurately applied one or more calibrated scale bar(s) of known material are measured. When the calibrated bar is made from the same material as the object the scaled measurements of its length should match the published calibration length.

Common metrology practice enforces the condition that the measured point to point distance must match the calibrated distance within  $\pm 0.05$  mm (0.002"). When the measurements and bar length match within the tolerance the measurement results pass this acceptance criteria.

The point to point measurements of the certified bar are used to confirm the thermal compensation function was correctly applied. The process uses the certified bar to check the object temperature measurement and CTE value.

## Propagation of Uncertainty

The propagation of uncertainty characterizes the probability and range of values within which the true value lies. It is used to reliably model the dependence of variable uncertainty on the uncertainty of the functions output. The variables in thermal compensation are object temperature, material CTE and the point to point distance measurement uncertainty.

The uncertainty for a distance can be bounded by the absolute error of the functions output. In this case the effect of the input variable uncertainties (or errors) on the thermal compensation function are studied. This uncertainty model is limited to the effect of thermal compensation.

The thermal compensation function/model and assumptions for an analysis example are shown below.

$$L_i = L_0(1 - \alpha\Delta T)$$

$$f(L_0, \alpha, \Delta T) = L_i$$

Uncertainty of  $L_i$  is a function of  $L_0, \sigma_L, \alpha, \sigma_\alpha, \Delta T, \sigma_T$

Example : 2 meter Alum Scale Bar from 10° to 40°C

$$L_0 = 2000 \text{ mm } \sigma_L = 0.02 \text{ mm}$$

$$\Delta T = -10 \dots 20 \text{ } ^\circ\text{C } \sigma_T = 0.5 \text{ } ^\circ\text{C}$$

$$\alpha_{alum} = 23.8 \text{ ppm}/^\circ\text{C } \sigma_\alpha = 5\% \alpha$$

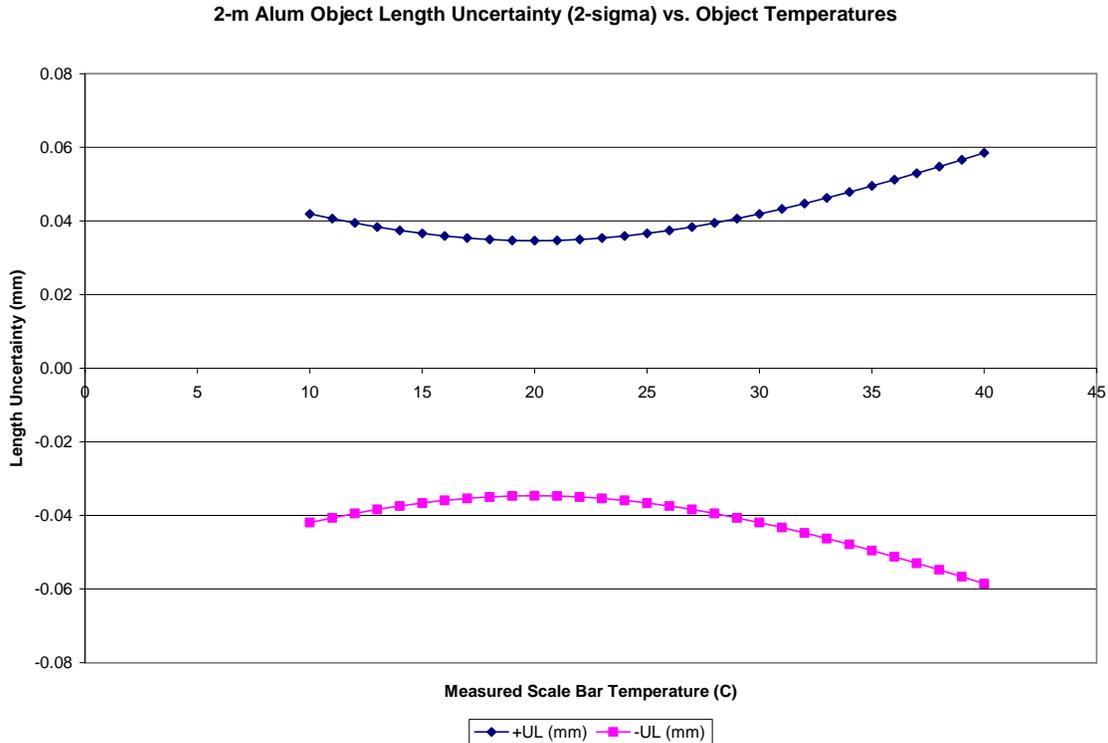
An equation for the variance between products by applying the Propagation of Uncertainty technique is shown below; it combines estimates from individual measurements.

$$\text{Model: } L_i = L_0(1 - \alpha\Delta T) = f(L_i)$$

$$U[f] = s_f = \sqrt{\left(\frac{\delta f}{\delta L_0}\right)^2 \sigma_{L_0}^2 + \left(\frac{\delta f}{\delta \alpha}\right)^2 \sigma_{\alpha}^2 + \left(\frac{\delta f}{\delta \Delta T}\right)^2 \sigma_T^2} \quad 1$$

$$U[L_i] = s_L = \sqrt{(1 - \alpha\Delta T)^2 \sigma_{L_0}^2 + (L_0\Delta T)^2 \sigma_{\alpha}^2 + (L_0\alpha)^2 \sigma_T^2}$$

A graph of the scale bar length uncertainty verse a range of object temperatures is shown in the chart.



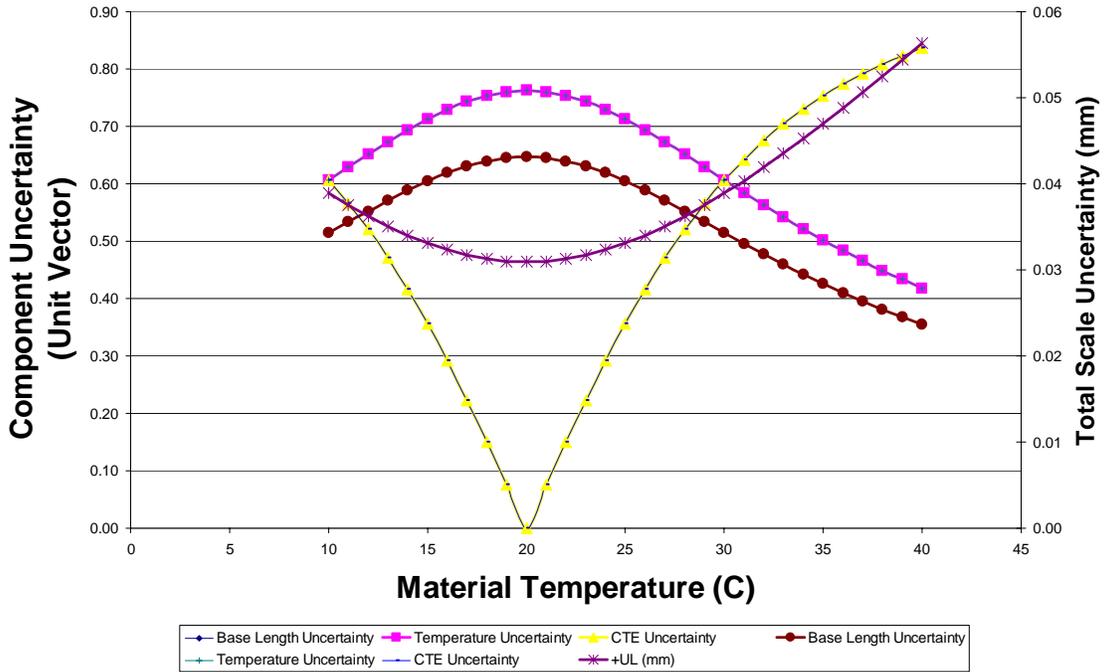
For this example a 2-meter (78.74”) aluminum scale bar certification certificate reports an uncertainty of 0.0254 mm (±0.001”) with k = 2. The uncertainty of the bar when used in the shop at the reference temperature is 0.035 mm (0.0014”). This number is bigger than the certification certificate due to effects of the temperature measurement uncertainty.

The uncertainty grows to 0.042 mm (0.0016”) when the delta temperature is 10 C (18 F). At a delta temperature of 20 C (36 F) the bars uncertainty grows to 0.059 mm (0.0023”). The difference is due to the effects of temperature measurement and CTE uncertainty.

Each of the individual uncertainty components is compared by computing a unit vector at each temperature. Each component’s relative contribute is shown in the scale length uncertainty components chart below.

<sup>1</sup> Leo Goodman (1960). "On the Exact Variance of Products" in *Journal of the American Statistical Association*, December, 1960, pp. 708-713.

### CTE Scaling Components of Unit Vector



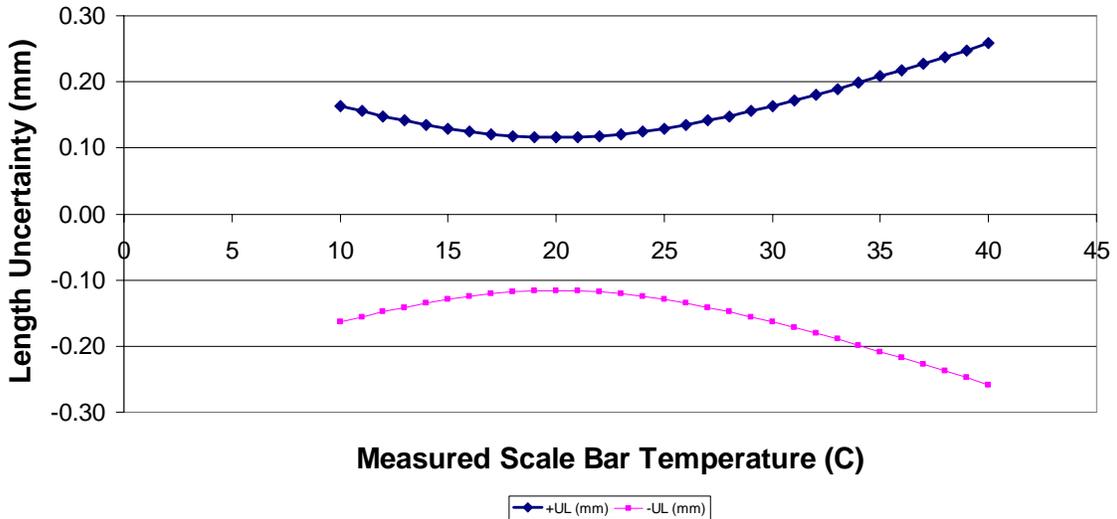
The chart shows that the CTE component has minimal effect when the object temperature is close to the reference temperature. However its contribution grows considerably as the object temperature deviates from the reference. The CTE component becomes the dominate uncertainty contributor past a delta of 10° C (18° F).

Common metrology systems (e.g., Laser Trackers) are able to make measurements on 2-meter long objects with uncertainties that are less than the thermal compensation uncertainties shown in the charts above. The implication is using the thermal compensation function to set the survey scale can add significant amounts of variability to measurement results. Variability increases as the object temperature deviates from the reference.

### Thermal Compensation Uncertainty Example

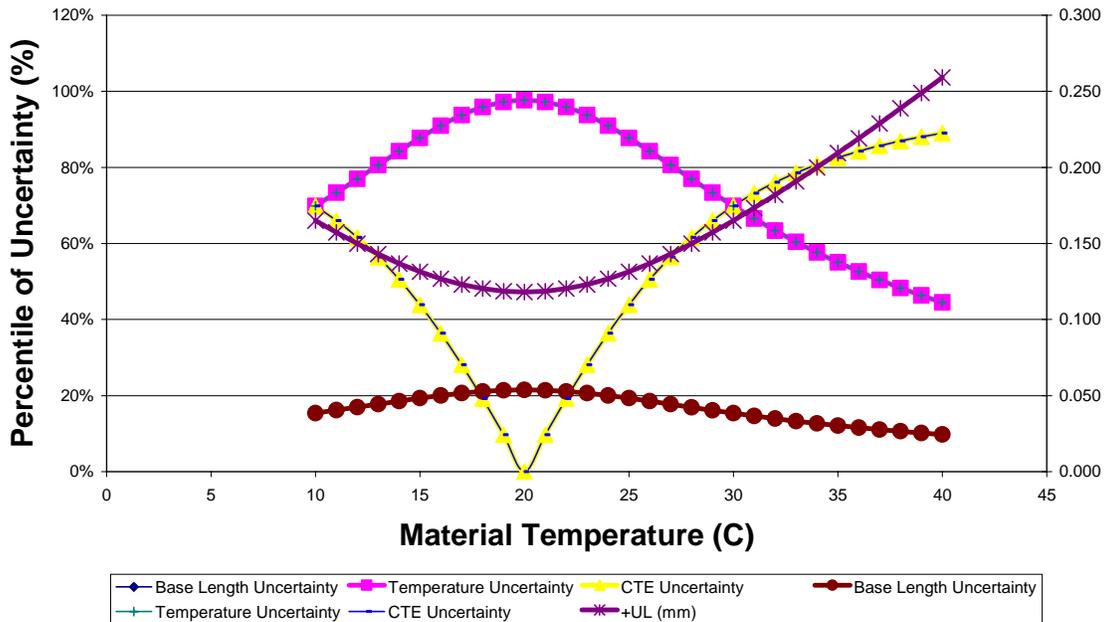
When the scale factor from thermal compensation process is applied to larger objects an the error gets magnified. The chart below shows the thermal compensation uncertainty for a 9.78-m (32-ft) aluminum object. At a delta temperature of 10 C (18 F) the uncertainty is 0.165 mm (0.0065”) with k=2.

### 9.78-m Alum Object Length Uncertainty due to Thermal Comp (2-sigma) vs. Object Temperature



These thermal compensation uncertainties are seen on the shop floor. The variation in measurement results are perceived as instrument or setup repeatability issues. The underlying property is related to precisely measuring the object’s core temperature relative to the reference and CTE variability.

### CTE Scaling Component Uncertainty Percentile (Aluminum Object 9.74 m)



The calibrated scale bar length uncertainty is a small component when the measuring larger objects. This result is apparent because an object temperature measurement error and CTE variation are multiplied onto a longer distance.

## **Solution ... Better Metrology Practice**

A better choice when measuring large objects is to use a like-kind material calibrated scale bar(s) to scale the measurements. A function to compute a scale factor that minimizes the differences between a series of measured scale bar distances (between the points and calibrated length) is straight-forward to implement. Applying the scale factor to the instrument results in temperature compensated measurement results without measuring the temperature or estimating the material's CTE. Error propagation analysis shows the object temperature measurement and CTE are significant uncertainty contributors.

Calibrated bar uncertainties are generally less than half the thermal compensation uncertainty on larger objects based on the error propagation model analysis. Reducing measurement process uncertainty by scaling with calibrated scale bars has other benefits.

Scaling with calibrated lengths integrates the traceable artifact into the measurement process. Current practices may use a traceable length to check thermal compensation results. Adding multiple bar positions enables different scaling at different vertical locations on the object. When measured into different groups or by different instrument stations a solution for local a scale factors would allow a consistent method to adjust for thermal compensation gradients. Different calibrated scale bar materials surveyed into different groups or by different instrument stations means a method to compensate for different thermal coefficients of expansions within the same survey.

Temperature measurement on the object can be used as a check/validation for the scaling with calibrated bars. In this case the method with lower uncertainty is used to set the scale factor while the less-precise method is used to check it.

## **Scale Lengths in USMN**

Scale lengths with uncertainty estimates can effectively be considered instruments within the network. Point to point distance residuals between observations and the calibrated distance are added as corrections to the network during the optimization. When weighted with their published uncertainty statement they constrain the resulting Composite Point group with appropriate weights. This method adds physical traceable length(s) to the network. Reports and instrument uncertainty analysis include direct comparison against these traceable standards.

The USMN solution integrates scale lengths as either "Report Only" or as "Constraints" in its optimization and target uncertainty analysis processes. As "Report Only" the scale bar do not add corrections to the network. The "Report Only" configuration is the default setup. In a "Report Only" configuration users can check or confirm the network's present thermal compensation. The figure below shows an example Measured Scale Bar report dialog when in "Report Only" mode.

PointA	PointB	Nominal	Actual	Delta
A::sb1::sb1_1	A::sb1::sb1_2	2438.38476	2438.29186	-0.09290
A::sb1::sb1_3	A::sb1::sb1_4	2438.38476	2438.29170	-0.09306
A::sb1::sb1_5	A::sb1::sb1_6	2438.38476	2438.29131	-0.09345
A::sb2::sb2_1	A::sb2::sb2_2	2438.38476	2438.28601	-0.09875
A::sb2::sb2_3	A::sb2::sb2_4	2438.38476	2438.28797	-0.09679
A::sb2::sb2_5	A::sb2::sb2_6	2438.38476	2438.28729	-0.09747
A::sb3::sb3_1	A::sb3::sb3_2	2438.38476	2438.14396	-0.24080
A::sb3::sb3_3	A::sb3::sb3_4	2438.38476	2438.14168	-0.24308
A::sb3::sb3_5	A::sb3::sb3_6	2438.38476	2438.13618	-0.24858
A::sb3::sb3_7	A::sb3::sb3_8	2438.38476	2438.14184	-0.24292

Measured Scale Bar Report Analysis against Certified Lengths (Report Only)

When the USMN uses measurements of scale length as constraints the network is adjusted along the components to minimize the delta between the length inputs. Each instrument station can be setup to allow its scale factor to be included in the network optimization. The figure above indicates Station 3 measurements have not been thermal compensated. Note the red outlined deltas for station 3 are higher. When instrument scale factors are included the result balances the network’s thermal compensation during the optimization. Reports of all the scale bar lengths show the resulting measurement deltas against the traceable standards.

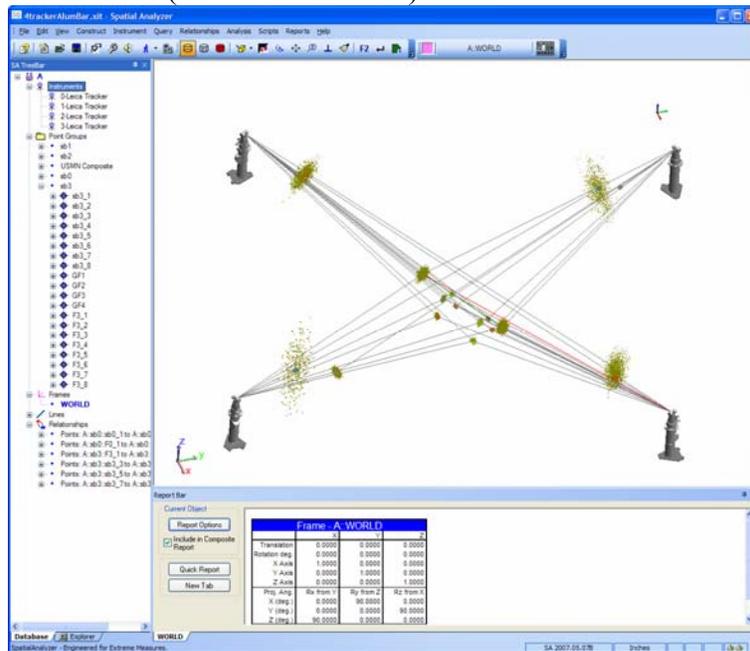
The figure below shows the Measured Scale Bar report after the multiple scale bars positions have been used as Constraints in the optimization.

PointA	PointB	Nominal	Actual	Delta
A::sb1::sb1_1	A::sb1::sb1_2	2438.38476	2438.37941	-0.00535
A::sb1::sb1_3	A::sb1::sb1_4	2438.38476	2438.37926	-0.00550
A::sb1::sb1_5	A::sb1::sb1_6	2438.38476	2438.37886	-0.00590
A::sb2::sb2_1	A::sb2::sb2_2	2438.38476	2438.37745	-0.00731
A::sb2::sb2_3	A::sb2::sb2_4	2438.38476	2438.37941	-0.00535
A::sb2::sb2_5	A::sb2::sb2_6	2438.38476	2438.37873	-0.00603
A::sb3::sb3_1	A::sb3::sb3_2	2438.38476	2438.38691	0.00215
A::sb3::sb3_3	A::sb3::sb3_4	2438.38476	2438.38464	-0.00012
A::sb3::sb3_5	A::sb3::sb3_6	2438.38476	2438.37914	-0.00562
A::sb3::sb3_7	A::sb3::sb3_8	2438.38476	2438.38480	0.00004

Measured Scale Bar Report Analysis against Certified Lengths (Constraints)

An example job using multiple positions of a certified aluminum scale bar is shown in the figure below. There were 4 tracker stations in this survey. The bar was made of Aluminum and was 2.44 m (96-inches) in length. Temperature measurements of the bar showed it was between 23 C and 25 C during the measurement process. The bar was measured in different 10 positions.

Two gage these effects observations were made on two points that were 9.87-m (32 ft) apart. The uncertainty with the thermal compensation function was at 22.5 C  $\pm$ 0.13 mm and at 23.5 C  $\pm$ 0.13 mm. In this case the published certified length standard was 2.44 m (96 inches) long with an uncertainty of  $\pm$ 0.02 mm (0.001 inches). The survey was scaled with thermal compensation function and then scaled using the scale bars. The 9.87 m point to point deltas were evaluated in both cases. The net difference between the results was 0.10 mm in the 9.87 m ( $\approx$   $\pm$ 0.005" in 386").



Survey Scaled in USMN 4 stations 10 scale bar positions

## Conclusion

Metrology systems in use in factory environments make accurate measurements on objects. A challenge for users is to reliably scale those measurements back to a reference temperature to compensate for object thermal expansion/contraction. The Propagation of Uncertainty analysis shows using the thermal compensation function to set the survey scale can add significant amounts of variability to measurement results. The variability increases as the object temperature deviates from the reference.

Scaling 3D metrology surveys with the thermal compensation function using object temperature and CTE estimates increases uncertainty. Scaling the same measurement networks with certified lengths reduces measurement uncertainty and enhances traceability reporting. Scale bar constraints should be weighted by their relative published uncertainties in the network optimization.

Measurement uncertainty analysis is enhanced when traceable length standards are included in the analysis and subsequent report. Target Uncertainty Field Analysis is improved by including traceable length standards.

The conclusion from these results indicates using calibrated scale lengths as instruments produces better measurement results.

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